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New calculations of α-decay half-lives by the Viola-Seaborg formula

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Abstract. The Viola-Seaborg formula is a well-known formula for α-decay half-lives of heavy nuclei. In this work we obtain new parameters of this formula through a least-square fit to even-even nuclei between $Z = 84$ and $Z = 110$ with N greater than 126. On average, the formula can reproduce the half-lives of heavy even-even nuclei within a factor of 1.3. The formula with new parameters works well for the superheavy region which is a hot topic of nuclear physics. The numerical results from the formula are compared with those from the cluster model.

PACS. 27.90.+b $A \geq 220$ – 23.60.+e Alpha decay – 21.10.Tg Lifetimes

1 Introduction

Alpha radioactivity was first explained in 1908 by Rutherford in terms of nuclear decay by emission of a ^{4}He nucleus. Later, α emission was found to be a very general mode of decay for the ground states of trans-lead nuclei. As early as 1911, Geiger and Nuttall showed the linear relationship between the logarithm of half-life and α-decay energy, which was then called the Geiger-Nuttall law. In 1928 the quantum theory of α-decay was presented by Gamov and by Condon and Gurney. This theory can account for the very sensitive dependence of half-life on decay energy and predict a relationship like the Geiger-Nuttall law. In 1966 Viola and Seaborg generalized the Geiger-Nuttall law with additional adjustable parameters and proposed a new relation about the logarithm of half-life, α-decay energy and charge number of the parent nucleus [1]. This relation was called the Viola-Seaborg formula, later. The calculated half-lives of α-decay by the formula agree well with the experimental data. So it has been widely used to calculate α-decay half-lives for many years [2]. However, the parameters of the Viola-Seaborg formula were proposed many years ago. In recent years many new data of α-decay have been observed by experimental physicists [3–5]. So it is interesting to investigate the validity of the Viola-Seaborg formula for new elements and to obtain new parameters of the formula. Here we ignore α-decay "fine structure" and just use the ground-to-ground Q-value and the overall half-life.

In this paper, a new set of parameters is obtained by a least-square fit to 64 even-even nuclei between $Z = 84$ and $Z = 110$ with N greater than 126. Generally, the formula with new parameters can reproduce the experimental half-lives of heavy and superheavy even-even nuclei very well. So it is valuable in identifying new elements. As to the hindrance factor caused by the odd nucleons, we can include its effect with a set of parameters. We follow Viola and Seaborg using a set of constants as hindrance factors, but revise their values. In this way the new parameters of the Viola-Seaborg formula are obtained and new calculations of the half-lives are carried out.

2 Empirical formulas

As early as 1911, Geiger and Nuttall showed that a good straight line was obtained by plotting the logarithm of the half-life against the reciprocal square root of decay energy, $Q_\alpha^{-1/2}$:

$$\log_{10} T_\alpha = A + B\, Q_\alpha^{-1/2}\,, \tag{1}$$

where A and B are Z-dependent coefficients to be determined from fitting the experimental data. Z is the charge number of the parent nucleus. T_α is the experimental half-life of α-decay which is related to the total half-life $T_{\rm tot}$ by

$$T_\alpha = T_{\rm tot} \,/\, R_\alpha\,, \tag{2}$$

where R_α is the branching ratio of α-decay. Q_α is the α-decay energy given by the difference of rest masses of the

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nuclei,

$$Q_\alpha = M(A,Z) - M(A-4,Z-2) - M(^4\mathrm{He}) . \quad (3)$$

In 1966 Viola and Seaborg generalized the Geiger-Nuttall law and proposed the Viola-Seaborg formula:

$$\log_{10} T_\alpha = (aZ+b)\,Q_\alpha^{-1/2} + (cZ+d) + h_{\log} , \quad (4)$$

where the parameters a, b, c and d are obtained through a least-square fit to even-even nuclei and $h_{\log}$ is the hindrance factor for nuclei with unpaired nucleons. In the above formula the unit of decay energy is MeV and that of the half-life is s. The calculated α-decay half-lives of even-even nuclei by this formula agree well with the experimental data. For odd-A and odd-odd nuclei, the decay often goes mainly to excited states where the orbital of the odd nucleon is unchanged. This is called "favored" decay. The decay rates to states where the odd nucleon changes its orbital are much more complicated to evaluate theoretically and depend only partly on the variation of the angular momentum of the odd nucleon [6].

In this paper, the experimental α-decay data are taken from the 2003 atomic-mass table by Audi *et al.*[7]. Through a least-square fit to the experimental data of 64 even-even nuclei (^{212}Po–^{270}Ds), we obtain a new set of parameters for the Viola-Seaborg formula. Their values are: $a = 1.64062$, $b = -8.54399$, $c = -0.19430$, $d = -33.9054$. As to odd nuclei (odd-A and odd-odd nuclei) we follow the treatment by Viola and Seaborg and select a set of constants as the hindrance factors, which are obtained through the least-square fits to odd nuclei. Their values are

$$h_{\log} = \begin{cases} 0, & Z \text{ even}, \ N \text{ even}, \\ 0.8937, & Z \text{ even}, \ N \text{ odd}, \\ 0.5720, & Z \text{ odd}, \ N \text{ even}, \\ 0.9380, & Z \text{ odd}, \ N \text{ odd}. \end{cases} \quad (5)$$

For even-even nuclei the Viola-Seaborg formula with the new parameters a, b, c and d can reproduce the half-lives very well. The maximum of the deviation between the calculated half-life and the experimental one is only a factor of 3 for even-even nuclei. For odd nuclei, the hindrance factors are obtained through the least-square fits. We will discuss this at the end of next section.

3 Numerical results and discussions

In this paper, we select 64 even-even nuclei between $Z = 84$ and $Z = 110$ with N greater than 126 to fit the parameters a, b, c and d of the Viola-Seaborg formula. The numerical results are shown in fig. 1. It can be seen that for nuclei with fixed neutron number N ($N = 128$–132) the deviations between the logarithm of experimental half-lives and that of calculated ones decrease when the charge number goes from 84 to 90. Similarly, the deviations decrease for a series of isotopes (from Po to Th) when the neutron number goes from 128 to 132. At the subshell of $N = 152$ the deviations are slightly larger than that of the neighboring nuclei. It is also interesting to note that there are discontinuities of α-decay rates around the nuclear shell or subshell of $N = 126$ and $N = 152$. For many nuclei, the experimental points are close to the theoretical ones. This means that the calculated half-lives agree well with the experimental ones for many nuclei.

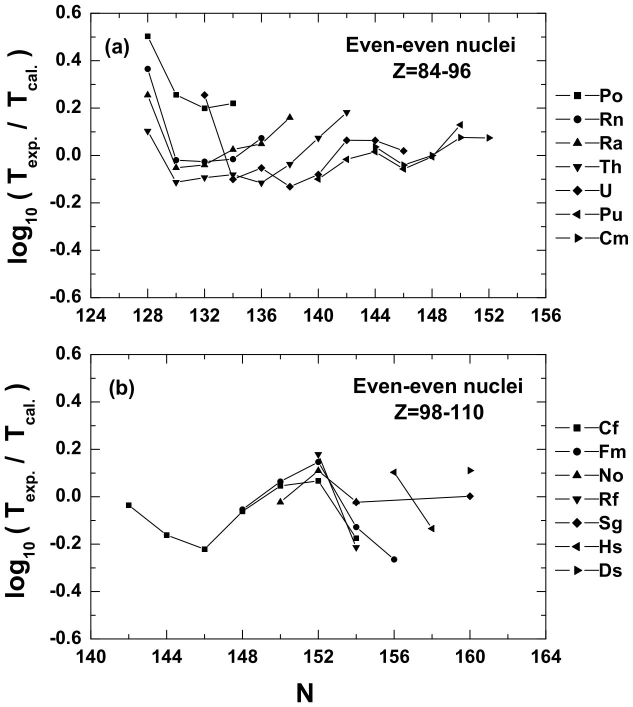

Fig. 1. The deviations between the experimental half-lives and the calculated ones of 64 even-even nuclei between $Z = 84$ and $Z = 110$ with N greater than 126.

Table 1. The mean deviations and the standard ones by the Viola-Seaborg formula with new parameters. The logarithms of the deviations 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 correspond to the deviations between the theoretical half-lives and experimental data by factors of 2, 2.5, 3.2, 4, 5 and 6.3, respectively.

Nuclei type	Number	$\sqrt{\sigma^2}$	$\langle\sigma\rangle$
even Z, even N	64	0.143	0.108
odd Z, odd N	15	0.787	0.617
even Z, odd N	42	0.580	0.488
odd Z, even N	33	0.396	0.321

In order to see clearly the agreement of the theoretical half-lives with the experimental data, we define the mean deviation and the standard deviation as follows.

The mean deviation is

$$\langle\sigma\rangle = \sum_{i=1}^{N} \left| \log_{10}\left(T^i_{\mathrm{exp.}} / T^i_{\mathrm{cal.}}\right) \right| / N . \quad (6)$$

The standard deviation is

$$\sqrt{\sigma^2} = \sqrt{\sum_{i=1}^{N} \left[\log_{10}\left(T^i_{\mathrm{exp.}} / T^i_{\mathrm{cal.}}\right) \right]^2 / N} . \quad (7)$$

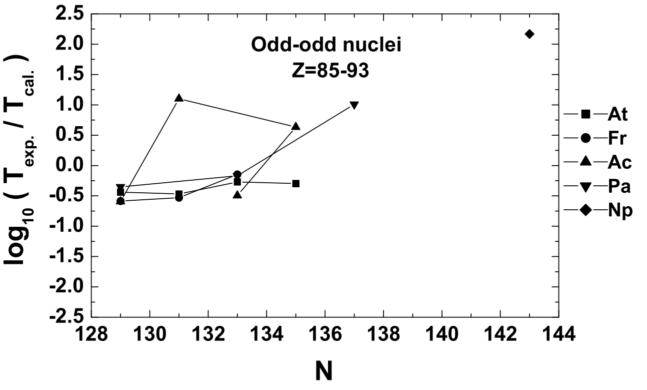

Fig. 2. The deviations between the experimental half-lives and the calculated ones of odd-odd nuclei.

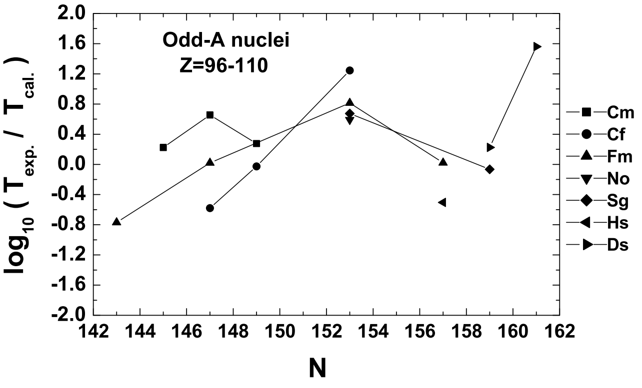

Fig. 4. The deviations between the experimental half-lives and the calculated ones of even-odd nuclei.

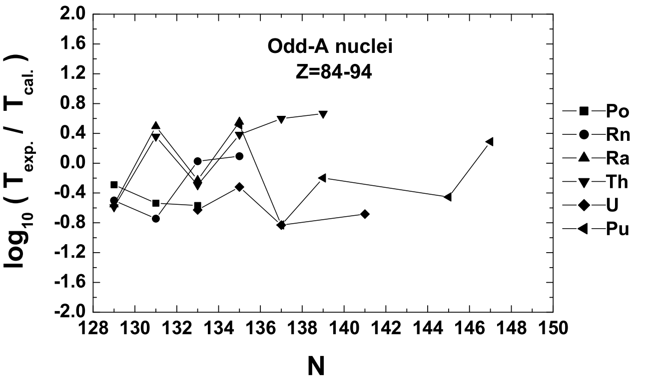

Fig. 3. The deviations between the experimental half-lives and the calculated ones of even-odd nuclei.

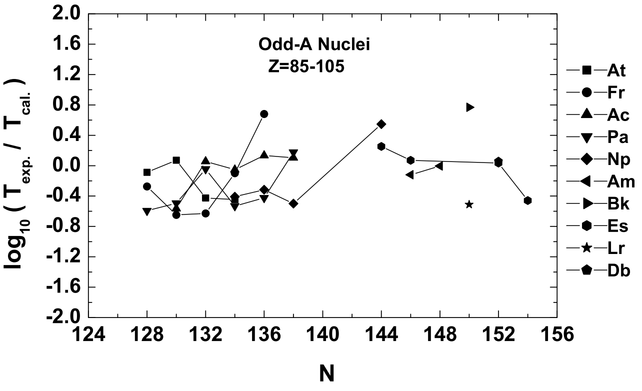

Fig. 5. The deviations between the experimental half-lives and the calculated ones of odd-even nuclei.

The values of the mean deviations and the standard deviations are listed in table 1. In table 1 the first column denotes the type of nuclei, the second column denotes the number of nuclei. The standard deviations are given in column 3 and the mean deviations are given in column 4. For even-even nuclei the mean deviation and the standard deviation are 0.143 and 0.108, respectively. This means that the Viola-Seaborg formula with the new parameters can reproduce the experimental data of even-even nuclei on average within a factor of 1.3. For odd nuclei the average deviations range from 0.321 to 0.617 (see table 1). This means that on average the half-life from the formula agrees with the data within a factor of 2–4 for odd nuclei. The mean deviations and the standard ones of odd-A nuclei are smaller than those of odd-odd nuclei. The numerical results are shown in figs. 2–5. It is also seen from figs. 2–5 that the deviations between theoretical half-lives and experimental ones are small for many nuclei. But the maximum deviations are the factors of 148, 36 and 6 for odd-odd, even-odd (even-Z, odd-N) and odd-even (odd-Z, even-N) nuclei, respectively. Because the hindrance factors in the formula represent the average effect of very complicated process of many nuclei, the deviation could be large for a single nucleus. Anyway, the agreement of the global behavior is acceptable.

4 Calculations of the half-lives of trans-fermium nuclei by the Viola-Seaborg formula with new parameters

The theoretical half-lives of 20 even-even nuclei from $Z = 100$ to $Z = 118$ are listed in table 2. The first column denotes the parent nuclei, the second column shows the α-decay energies (in MeV). The logarithms of the experimental half-lives (in s) and those of the calculated ones are given in column 3 and column 4, respectively. The last column gives calculated half-lives by the cluster model [8,9]. Except for $^{292}116$, theoretical half-lives agree with experimental data within a factor of 2. The standard deviation is 0.2 and the mean deviation is 0.15. This means that, on average, the Viola-Seaborg formula with new parameters can reproduce the half-lives of heavy nuclei (between $Z = 100$ and $Z = 118$) within a factor of 1.4. For $^{292}116$ the logarithm of the deviation is 0.611 (corresponding to a factor of 4), and it is also reasonable. It means that the formula is valid in this mass region. It is also seen that the theoretical half-lives from the formula are very close to those from the cluster model, although the two models are different in mechanism.

Table 2. The logarithms of half-lives of 20 even-even nuclei calculated from the Viola-Seaborg formula with new parameters. The experimental data from Fm to Ds are taken from the 2003 atomic-mass table by Audi *et al.* [7]. The data of the last five nuclei ($Z = 114$–118) are taken from ref. [5].

Nuclei	Q_α (MeV)	$T_{\rm exp.}$	$T_{\rm cal.}^{\rm V\text{-}S}$	$T_{\rm cal.}^{\rm CM}$ [ref.]
^{248}Fm	8.002	1.588	1.642	1.633 [8]
^{250}Fm	7.557	3.301	3.237	3.230 [8]
^{252}Fm	7.153	4.961	4.814	4.806 [8]
^{254}Fm	7.308	4.067	4.195	4.204 [8]
^{256}Fm	7.027	5.067	5.332	5.342 [8]
^{252}No	8.550	0.561	0.584	0.568 [8]
^{254}No	8.226	1.753	1.643	1.643 [8]
^{256}No	8.581	0.464	0.486	0.568 [8]
^{256}Rf	8.930	0.304	0.126	0.079 [8]
^{258}Rf	9.250	−1.035	−0.821	−0.678 [9]
^{260}Sg	9.920	−2.022	−1.999	−1.959 [8]
^{266}Sg	8.630	1.791	1.789	1.276 [9]
^{264}Hs	10.59	−2.966	−3.070	−2.859 [9]
^{266}Hs	10.34	−2.569	−2.434	−2.347 [9]
^{270}Ds	11.20	−3.796	−3.906	−3.740 [9]
^{286}114	10.35	−0.398	−0.576	−0.333 [9]
^{288}114	10.09	−0.097	0.135	0.544 [9]
^{290}116	11.00	−1.824	−1.639	−1.004 [9]
^{292}116	10.80	−1.745	−1.134	−0.742 [9]
^{294}118	11.81	−2.745	−2.986	−2.620 [9]

5 Conclusions

Through a least-square fit we obtain a new set of parameters for the Viola-Seaborg formula. The formula with this set of parameters can reproduce the half-lives of heavy and superheavy even-even nuclei ($Z = 84$–118) very well. On average, the theoretical half-lives agree with the experimental data within a factor of 1.4 for even-even nuclei with $Z = 84$–184. For odd nuclei, the theoretical half-lives agree reasonably with the data in global behavior although somewhat large deviations exist for a few nuclei. The formula with new parameters is useful for quick estimates of α-decay half-lives of future superheavy experiments.

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